NON-SINGULAR COSMOLOGY IN MODIFIED GRAVITY

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Abstract

A non-singular cosmology is derived in modified gravity (MOG). The universe begins in the distant past in a contracting de Sitter phase, is non-singular at $t = 0$ and then describes approximately the standard radiation dominated solution before the time of decoupling. The spacetime in the neighborhood of $t = 0$ is described by Minkowski spacetime in which the Weyl curvature tensor vanishes and the entropy is at a minimum value. The Hubble radius $H^{-1}(t)$ is infinite at $t = 0$ and the universe goes into a static or quasi-static period, and the temperature of the hot radiation plasma at $t \sim 0$ does not exceed the Hagedorn temperature.

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1 Introduction

The singularity in standard cosmology at $t = 0$ heralds the break down of physics at the big bang. This has been a long-standing problem in cosmology that prevents a rational explanation for the onset of the beginning of the universe. We know from the afterglow of the cosmic microwave background (CMB) radiation with the uniform temperature $\sim 2.73^0\,K$ that according to the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology there had to be a hot beginning to the universe. In the following, we shall develop a non-singular cosmology using the effective classical action and field equations based on modified gravity (MOG) [1, 2].

An initial value problem occurs at the beginning of the universe at the big bang $t = 0$, arising from one of the most sacred laws of physics: the Second Law of Thermodynamics. This problem has been addressed by Penrose [3, 4], who has proposed a solution based on making the Weyl curvature tensor vanish at $t = 0$.

He also proposed that the universe went through a series of conformally invariant cycles [5].

The problem with the second law of thermodynamics arises because the entropy of the universe increases with time, and accordingly the disorder or lack of speciality increases as the universe expands. The entropy must have decreased with time as we approach the big bang at $t = 0$, for it increases as time increases and we do not wish to contemplate a violation of the second law of thermodynamics, which is as cherished as the law of conservation of energy. Therefore, the initial state of the universe was the most special state of all. Because of the singularity at $t = 0$ in big bang cosmology, the entropy at the big bang in the phase space available is huge contradicting the Second Law of Thermodynamics.

In inflationary models it is assumed that inflation began in a homogeneous patch in a "random" state some short time after the big bang. However, the randomness of this initial state is the most extreme non-special state and therefore inflation does not solve the entropy problem. In variable speed of light (VSL) cosmology the large value of the speed of light achieved for a short period of time in the early universe can lead to a huge decrease in the initial entropy [6, 7, 8]. After the speed of light returns to its currently measured value through a phase transition, the entropy increases enormously and sets the direction of the arrow of time when a particular direction of time is chosen by the choice of a spontaneous breaking of Lorentz invariance. The VSL model also resolves the horizon problem. However, we are still left with the problem of the occurrence of an initial singularity in inflationary and VSL models.

In the following, we shall pursue a non-singular cosmological model which can resolve initial value problems such as the horizon problem and the entropy conundrum. We shall assume that quantum gravity can be described as an asymptotically free theory leading to a non-perturbatively renormalizable theory and the "running" of the gravitational constant with momentum, $G = G(k)$ [9, 10, 11]. By means of an effective gravitational action the running of G with momentum can be converted to a variation of G with distance and the asymptotic freedom of quantum gravity demands that G vanishes at zero space and time $|x| \to 0$.

Lacking a fully developed asymptotically free and renormalization group flow quantum gravity, we shall study the MOG cosmology using the effective classical scalar-tensor-vector gravity (STVG) [1, 2]. This MOG with its variations of G, the vector field ϕ_μ, the coupling strength ω and its effective mass μ leads to a satisfactory description of galaxy rotation curves, the mass profiles of X-ray clusters of galaxies [12, 13], the bullet cluster 1E0657-56 [14] and to the WMAP data for the CMB [2] without undetected dark matter.

2 MOG Field Equations

The gravitational field equations are given by [1, 2]:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda + Q_{\mu\nu} = 8\pi G_N \xi T_{\mu\nu}, \tag{1}$$

where $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R$ and $\xi(x)$ denotes the scalar field describing the variation of the gravitational "constant", $G = G_N\xi(x)$ where G_N denotes Newton's constant. We have chosen units with $c = 1$ and ∇_μ denotes the covariant derivative with respect to the metric $g_{\mu\nu}$. We adopt the metric signature $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ where $\eta_{\mu\nu}$ is the Minkowski spacetime metric, $R = g^{\mu\nu}R_{\mu\nu}$ and Λ denotes the cosmological constant.

We have

$$Q_{\mu\nu} = \xi\biggl[\nabla^\alpha\nabla_\alpha\biggl(\frac{1}{\xi}\biggr)g_{\mu\nu} - \nabla_\mu\nabla_\nu\biggl(\frac{1}{\xi}\biggr)\biggr], \tag{2}$$

The quantity $Q_{\mu\nu}$ results from a boundary contribution arising from the presence of second derivatives of the metric tensor in R in the action. These boundary contributions are equivalent to those that occur in Brans-Dicke gravity theory [15].

In the original formulation of MOG [1], the vector field ϕ_μ satisfies massive Maxwell-Proca-type field equations. We shall generalize these field equations to massive Yang-Mills field equations of the form:

$$D_\nu B^{a\mu\nu} + \frac{\partial V(\phi)}{\partial\phi^a_\mu} + \frac{1}{\omega}\Delta_\nu\omega B^{a\mu\nu} = -\frac{1}{\omega}J^{a\mu}. \tag{3}$$

Here, $B^{a\mu\nu}$ denotes the Yang-Mills field:

$$B^{a\mu\nu} = \partial_\mu\phi^a_\nu - \partial_\nu\phi^a_\mu + C^{abc}\phi^b_\mu\phi^c_\nu, \tag{4}$$

and we have included contributions from the variation of the effective Yang-Mills coupling strength ω. Moreover, D_μ denotes the Yang-Mills covariant derivative acting on a Dirac field $\Psi(x)$:

$$D_\mu\Psi(x) = [\partial_\mu + \phi_\mu(x)]\Psi(x), \tag{5}$$

where

$$\phi_\mu(x) = -i\omega\phi^a_\mu(x)T^a, \quad B^{\mu\nu}(x) = -i\omega B^{a\mu\nu}(x)T^a. \tag{6}$$

Here, T^a are the matrix representations of the generators of the $U(n)$ symmetry group.

In the absence of contributions from the potential $V(\phi)$ the action for the Yang-Mills field ϕ_μ is invariant under the gauge transformations

$$\phi^{'}_\mu(x) = U(x)\phi_\mu(x)U^{-1}(x) - [\partial_\mu U(x)]U^{-1}(x). \tag{7}$$

However, in general there will be contributions in the potential $V(\phi)$ from a mass term $-(1/2)\mu^2\phi^\mu\phi_\mu$ which will break the gauge symmetry. This mass term can be obtained from a spontaneous symmetry breaking of the action.

The current conservation law is now of the form

$$D_\mu J^{a\mu} = 0. \tag{8}$$

The non-abelian charge associated with the fermion current density is no longer a constant of the motion, for the gauge field ϕ_μ is no longer neutral with respect to the "fifth force" charge and their is a non-linear coupling between the ϕ_μ fields, leading to a non-zero ϕ_μ charge current density. This means that the fermions carrying the non-abelian fifth force charge obey a non-linear relation between the charges. In the Maxwell-Proca version of MOG, which is invariant under $U(1)$ abelian gauge transformations for massless ϕ_μ fields, the fermion fifth force point charges add linearly. The non-linear relation between fermion point charges in the Yang-Mills description of the fifth force can have important implications for the explanation of astrophysical data [12, 13].

The total energy-momentum tensor is given by

$$T_{\mu\nu} = T_{M\mu\nu} + T_{\phi\mu\nu} + T_{S\mu\nu}, \tag{9}$$

where $T_{M\mu\nu}$, $T_{\phi\mu\nu}$ and $T_{S\mu\nu}$ denote the energy-momentum tensor contributions of ordinary matter, the ϕ_μ field and the scalar fields ξ, ω and μ, respectively. We have

$$T_{\phi\mu\nu} = \omega\biggl[B_\mu{}^\alpha B_{\nu\alpha} - g_{\mu\nu}\biggl(\frac{1}{4}B^{\rho\sigma}B_{\rho\sigma} + V(\phi)\biggr) + 2\frac{\partial V(\phi)}{\partial g^{\mu\nu}}\biggr]. \tag{10}$$

The $\xi(x)$ field yields the energy-momentum tensor:

$$T_{\xi\mu\nu} = -\frac{1}{G_N\xi^3}\biggl[\nabla_\mu\xi\nabla_\nu\xi - 2\frac{\partial V(\xi)}{\partial g^{\mu\nu}} - g_{\mu\nu}\biggl(\frac{1}{2}\nabla_\alpha\xi\nabla^\alpha\xi - V(\xi)\biggr)\biggr], \tag{11}$$

where $V(\xi)$ denotes a potential for the scalar field ξ.

From the Bianchi identities

$$\nabla_\nu G^{\mu\nu} = 0, \tag{12}$$

and from the field equations (1), we obtain

$$\nabla_\nu T^{\mu\nu} + \frac{1}{\xi}\nabla_\nu\xi T^{\mu\nu} - \frac{1}{8\pi G_N\xi}\nabla_\nu Q^{\mu\nu} = 0. \tag{13}$$

The scalar field $\xi(x)$ satisfies the field equations

$$\nabla_\alpha\nabla^\alpha\xi + V'(\xi) + N(\xi) = \frac{1}{2}G_N\xi^2\biggl(T + \frac{\Lambda}{4\pi G_N\xi}\biggr), \tag{14}$$

where

$$N(\xi) = -\frac{3}{\xi}\biggl(\nabla_\alpha\xi\nabla^\alpha\xi + V(\xi)\biggr) + \frac{1}{16\pi}\xi^2\nabla_\alpha\nabla^\alpha\biggl(\frac{1}{\xi}\biggr)$$
$$+\xi\biggl(\frac{1}{2}\nabla_\alpha\omega\nabla^\alpha\omega - V(\omega)\biggr) + \frac{\xi}{\mu^2}\biggl(\frac{1}{2}\nabla_\alpha\mu\nabla^\alpha\mu - V(\mu)\biggr), \tag{15}$$

and $T = g^{\mu\nu}T_{\mu\nu}$. Similar field equations hold for the scalar fields $\omega(x)$ and $\mu(x)$ [1].

We observe that our field equations (14) for the variation of G contain a potential $V(\xi)$, which is absent in standard Brans-Dicke gravity [15]. Moreover, the conservation of energy equation (13) is more general than in Brans-Dicke gravity in which $\nabla_\nu T_M^{\mu\nu}=0$ is imposed from the outset.

The trace free Weyl curvature tensor is defined by

$$C_{\mu\nu\rho\sigma}=R_{\mu\nu\rho\sigma}-g_{\mu[\rho}R_{\sigma]\nu}-g_{\nu[\rho}R_{\sigma]\mu}+\frac{1}{3}Rg_{\mu[\sigma}g_{\rho]\nu}. \tag{16}$$

Under a conformal transformation of the metric:

$$\tilde{g}^{\mu\nu}=\Omega^{-2}g^{\mu\nu}, \tag{17}$$

the Weyl tensor is unchanged:

$$\tilde{C}^{\sigma}_{\mu\nu\rho}=C^{\sigma}_{\mu\nu\rho}. \tag{18}$$

The field equations (1) are not conformally invariant due to the non-vanishing trace of the energy-momentum tensor, $T\neq 0$.

3 Non-Singular Cosmology

Let us now consider a cosmological solution to our MOG theory. We adopt a homogeneous and isotropic FLRW background geometry with the line element

$$ds^2=dt^2-a^2(t)\left(\frac{dr^2}{1-kr^2}+r^2d\Omega^2\right), \tag{19}$$

where $d\Omega^2=d\theta^2+\sin^2\theta d\phi^2$ and $k=0,-1,+1$ for a spatially flat, open and closed universe, respectively. Due to the symmetry of the FLRW background spacetime, we have $\phi_0\neq 0$, $\phi_i=0$ and $B_{\mu\nu}=0$.

We define the energy-momentum tensor for a perfect fluid by

$$T^{\mu\nu}=(\rho+p)u^\mu u^\nu-pg^{\mu\nu}, \tag{20}$$

where $u^\mu=dx^\mu/ds$ is the 4-velocity of a fluid element and $g_{\mu\nu}u^\mu u^\nu=1$. Moreover, we have

$$\rho=\rho_M+\rho_\phi+\rho_S,\quad p=p_M+p_\phi+p_S, \tag{21}$$

where ρ_i and p_i denote the components of density and pressure associated with the matter, the ϕ^μ field and the scalar fields ξ, ω and μ, respectively.

The modified Friedmann equations take the form [1, 2]:

$$\frac{\dot{a}^2(t)}{a^2(t)}+\frac{k}{a^2(t)}=\frac{8\pi G_N\xi(t)\rho(t)}{3}+f(t)+\frac{\Lambda}{3}, \tag{22}$$

$$\frac{\ddot{a}(t)}{a(t)}=-\frac{4\pi G_N\xi(t)}{3}[\rho(t)+3p(t)]+h(t)+\frac{\Lambda}{3}, \tag{23}$$

where $\dot{a} = da/dt$ and

$$f(t) = \frac{\dot{a}(t)}{a(t)}\frac{\dot{\xi}(t)}{\xi(t)}, \tag{24}$$

$$h(t) = \frac{1}{2}\left(\frac{\ddot{\xi}(t)}{\xi(t)} - \frac{\dot{\xi}^2(t)}{\xi^2(t)} + 2\frac{\dot{a}(t)}{a(t)}\frac{\dot{\xi}(t)}{\xi(t)}\right). \tag{25}$$

The conservation law for matter is given by

$$\dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) + \mathcal{I} = 0, \tag{26}$$

where

$$\mathcal{I} = \frac{3a^2}{8\pi G_N \xi}(2\dot{a}f + a\dot{f} - 2\dot{a}h). \tag{27}$$

We shall now impose the approximate conditions:

$$2\dot{a}f + a\dot{f} \sim 2\dot{a}h, \tag{28}$$

$$\frac{d}{dt}\left(\frac{\dot{\xi}}{\xi}\right) < 2\frac{\dot{a}}{a}\frac{\dot{\xi}}{\xi}. \tag{29}$$

We find from (25) and (29) that $f \sim h$, and from the condition (28) we obtain

$$\dot{f} \equiv \frac{d\Lambda_G}{dt} \sim 0, \tag{30}$$

where

$$\Lambda_G = \frac{\dot{a}}{a}\frac{\dot{\xi}}{\xi}. \tag{31}$$

By setting the cosmological constant $\Lambda = 0$, we get the generalized Friedmann equations

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{8\pi G_N \xi \rho}{3} + \Lambda_G, \tag{32}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N \xi}{3}(\rho + 3p) + \Lambda_G. \tag{33}$$

We now have from (26), (27) and (28) that $\mathcal{I} \sim 0$ and

$$\dot{\rho} + 3\frac{d\ln a}{dt}(\rho + p) \sim 0. \tag{34}$$

We adopt the equation of state: $p(t) = w\rho(t)$ and derive from (34) the approximate solution for $\rho(t)$:

$$\rho(t) \sim \rho_0\left(\frac{a_0}{a(t)}\right)^{3(1+w)}, \tag{35}$$

where t_0 denotes the present time, $\rho_0 = \rho(t_0)$, $a_0 = a(t_0)$, $a/a_0 = 1/(1+z)$ and z denotes the red shift. For the matter and radiation densities ρ_m and ρ_r, we have $w = 0$ and $w = 1/3$, respectively. This gives

$$\rho_m(t) \sim \rho_{0m}(1+z)^3, \quad \rho_r(t) \sim \rho_{0r}(1+z)^4. \tag{36}$$

Let us make the simplifying approximation for the equations (14) with $\Lambda = 0$:

$$\ddot{\xi} + 3H\dot{\xi} + V'(\xi) = \frac{1}{2}G_N\xi^2(\rho - 3p). \tag{37}$$

A solution for ξ in terms of a given potential $V(\xi)$ and for given values of ρ and p can be obtained from (37). Previously a solution for $G(t) = G_N\xi(t)$ was obtained valid for big bang nucleosynthesis when $\xi \sim 1$ and for the period of recombination and the CMB [1, 2].

We shall be concerned in what follows with a solution valid from $t = -\infty$ to $t \geq 0$:

$$\xi(t) = (1 - \exp(-t/t_c))^2, \tag{38}$$

where t_c is a constant. We have $\xi(0) = 0$, $\xi(t) \to 1$ as $t \to \infty$ and $\xi(t) \to \exp(2|t|/t_c)$ as $t \to -\infty$. Moreover, we obtain

$$\dot{\xi}(t) = \biggl(\frac{2}{t_c}\biggr)\exp(-t/t_c)(1 - \exp(-t/t_c)) = \biggl(\frac{2}{t_c}\biggr)(1 - \xi^{1/2})\xi^{1/2}, \tag{39}$$

and

$$\ddot{\xi}(t) = -\biggl(\frac{2}{t_c^2}\biggr)\exp(-t/t_c)(1 - 2\exp(-t/t_c)) = \biggl(\frac{2}{t_c^2}\biggr)(1 - \xi^{1/2})(1 - 2\xi^{1/2}). \tag{40}$$

We can solve for t from (38) to give

$$t = t_c \ln\biggl(\frac{1}{1 - \xi^{1/2}(t)}\biggr). \tag{41}$$

The potential $V(\xi)$ as a function of ξ is given by

$$V(\xi) = -\biggl(\frac{2}{t_c^2}\biggr)\biggl\{\xi + \xi^2 - 2\xi^{3/2} + \biggl(\frac{G_N t_c^2}{12}\biggr)\xi^3(\rho - 3p)$$
$$+3\int d\xi\biggl[\frac{\xi(1 - \xi^{1/2})}{1 - \xi^{1/2} - \ln(1 - \xi^{1/2})}\biggr]\biggr\}. \tag{42}$$

For the radiation dominated era, $p = \frac{1}{3}\rho$, we have

$$V(\xi) = -\biggl(\frac{2}{t_c^2}\biggr)\biggl\{\xi + \xi^2 - 2\xi^{3/2} + 3\int d\xi\biggl[\frac{\xi(1 - \xi^{1/2})}{1 - \xi^{1/2} - \ln(1 - \xi^{1/2})}\biggr]\biggr\}. \tag{43}$$

By neglecting Λ_G in (32) and restricting ourselves to the radiation dominated epoch near $t = 0$, we get

$$\dot{a}(t) = \left[\frac{8\pi G_N \rho_0 \xi(t)}{3a^2(t)} - k\right]^{1/2}. \tag{44}$$

We choose the flat space version of (44) with $k = 0$ and integrating we obtain

$$a(t) = C^{1/4}\left(\int dt \xi^{1/2}(t)\right)^{1/2}, \tag{45}$$

where

$$C = \frac{32\pi G_N \rho_0}{3}. \tag{46}$$

Substituting (38) into (45) gives

$$a(t) = C^{1/4}(t + t_c \exp(-t/t_c))^{1/2}. \tag{47}$$

We see that as $t \to -\infty$ the solution (47) has the de Sitter form:

$$a(t) \sim C^{1/4} t_c^{1/2} \exp(|t|/2t_c), \tag{48}$$

while for $t = 0$ we obtain the finite result:

$$a(0) = C^{1/4} t_c^{1/2}. \tag{49}$$

The density of matter and radiation ρ and the temperature T are finite at $t = 0$. For $t \to \infty$, we regain the standard solution for an FLRW radiation dominated universe valid within our approximation scheme:

$$a(t) \sim C^{1/4} t^{1/2}. \tag{50}$$

At $t = 0$ spacetime is described by the conformally flat Minkowski metric:

$$ds^2 = dt^2 - a^2(0)(dx^2 + dy^2 + dz^2). \tag{51}$$

The Hawking-Penrose theorems [16, 17, 18] prove that a singularity must occur at $t = 0$ in classical GR when the weak and strong energy conditions are satisfied. The weak and strong energy conditions are satisfied when

$$T_{\mu\nu} U^\mu U^\nu \geq 0, \tag{52}$$

and

$$(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T) U^\mu U^\nu \geq 0, \tag{53}$$

where U^μ is a timelike or null vector. The conditions (52) and (53) hold when $\rho \geq 0$ and $\rho + 3p \geq 0$, respectively. From Eqs.(32) and (47) for $k = 0$, it follows that the weak energy condition $\rho > 0$ is satisfied. On the other hand, from Eqs.(33) and (47) in the absence of Λ_G, we find that

$$\rho + 3p = \frac{3}{4\pi G_N \xi}\biggl[\frac{\frac{1}{2}t_c(1-\exp(-t/t_c)^2 - \exp(-t/t_c)(t + t_c\exp(-t/t_c))}{t_c(t+t_c\exp(-t/t_c))^2}\biggr]. \tag{54}$$

We see for $t < t_c$ that $\rho + 3p$ can be negative violating the strong energy condition. We conclude from this that our non-singular cosmological solution is consistent with the Hawking-Penrose theorems, for the violation of the strong energy condition invalidates the no-go singularity theorems.

4 Evolution of Non-Singular Cosmology

Our cosmological solution derived from our MOG satisfies a non-singular solution in which the universe begins with a contracting de Sitter inflationary phase in the infinitely distant past, passes through $t = 0$ with $G(0) = 0$ without suffering a singular behavior and then satisfies the approximate radiation dominated solution of the FLRW type for $t > 0$. As the universe expands beyond the epoch of nuclear synthesis at a red shift $z \sim 10^9$, the solution for $G(t) = G_N\xi(t)$ obtained in Refs. [1, 2] takes over and leads to a description of the CMB at the surface of last scattering.

The spacetime neighborhood $\mathcal{M}_M$ at the time $t = 0$ is a special smooth region in which the Weyl curvature tensor defined in (16) vanishes: $C_{\mu\nu\rho\sigma} = 0$. Although the Weyl curvature vanishes for an FLRW spacetime metric, the vanishing of the Weyl curvature tensor at $t = 0$ is rigorous and can solve the Penrose conundrum [3], for the gravitational degrees of freedom vanish in $\mathcal{M}_M$ and the entropy of the universe S is at a minimum. As $G(t)$ grows monotonically from zero as t increases, the entropy will increase and obey the second law of thermodynamics as the universe expands.

The Hubble parameter for our solution is given by

$$H(t) \equiv \frac{\dot{a}(t)}{a(t)} = \frac{1-\exp(-t/t_c)}{2(t+t_c\exp(-t/t_c))}. \tag{55}$$

We see that $H(0) = 0$ and the universe stops expanding at $t = 0$. The epoch at $t \geq 0$ when the universe begins to expand consists of a hot radiation plasma and due to the red shifting of the radiation predicts the observed uniform temperature of the CMB. However, there is no singular big bang, for the universe spends an infinite amount of time before $t = 0$ initially in a contracting de Sitter phase and then passes *smoothly* through $t = 0$ leading to the current universe. In contrast to the standard big bang picture, the density of matter and radiation ρ and the temperature T are finite at $t = 0$. There is ample time for the growth of fluctuations producing galaxies and stars and this can be achieved with normal baryonic matter; there is no need for the undetected dark matter.

The proper particle horizon size is given by

$$d_H(t) = a(t)\int_{-\infty}^{t}\frac{dt'}{a(t')} = (t+t_c\exp(-t/t_c))^{1/2}\int_{-\infty}^{t}\frac{dt'}{t'+t_c\exp(-t/t_c)^{1/2}}. \tag{56}$$

It is exponentially increasing as $t \to -\infty$ and for $t \to \infty$ it describes a contracting de Sitter phase. The horizon size $d_H(t)$ can be big enough as the universe evolves from $t = -\infty$ to allow photons to be thermalized through repeated particle collisions and there is no problem with causality, predicting the uniform CMB temperature. Although scalar-field driven inflation has been successful in predicting a scale-invariant spectrum of adiabatic cosmological perturbations, it suffers from the problem of the initial cosmological singularity at $t = 0$. There have been recent articles using string theory to provide a non-singular cosmology [19]. Going backwards in time, the universe contracts and the temperature increases but does not exceed the Hagedorn temperature [20].

We adopt a cosmological scenario in which the universe is quasi-static for $t \sim 0$ and there exists a hot and dense phase at and after $t = 0$ in thermal equilibrium at a temperature close to the Hagedorn temperature. The Hubble radius $r_H = H^{-1}(t)$ is infinite or nearly infinite for $t \sim 0$. We can adopt a similar argument given by Nayeri, Brandenberger and Vafa [19] that for $t \sim 0$ the universe is in a static or quasi-static epoch and becomes radiation dominated as t increases from $t = 0$. The fluctuation modes k are inside the Hubble radius as $H(t)$ grows from $t = 0$ and they exit the Hubble radius $r_H = H^{-1}(t)$ at times $t_i(k)$ and then reenter the Hubble radius at late times $t_f(k)$. The initial collapsing de Sitter period in the distant past and the infinite Hubble radius $r_H = H^{-1}$ at $t = 0$ make it possible to produce fluctuations using causal physics.

5 Conclusions

We have obtained from MOG a smooth description of the large scale structure of spacetime. As the universe evolves from $t = -\infty$ there are no cosmological singularities and there is no "big bang" at the putative origin of the universe at $t = 0$. This universe can be compared with the steady-state universe of Hoyle, Bondi and Gold [21, 22] which has no big bang and is described by a de Sitter cosmological model with the continuous creation of matter. However, in the present model of the universe, we do not postulate a continuous creation of matter that violates the conservation of energy and our model is dynamically evolving and does not satisfy the perfect cosmological principle. Conserved matter and radiation are present throughout the evolution of the universe. In order for our scenario to agree with presently available cosmological observations, we postulate a hot radiation plasma at the epoch of $t \sim 0$ and the subsequent expansion of the universe has a similar evolution as in the big bang model. The CMB WMAP data and acceleration of the universe at a late-time epoch are described by MOG without dark matter and without a standard cosmological constant Λ [2].

We have seen that it is possible in the singularity-free MOG cosmology to solve the conundrum of the entropy problem, for at $t = 0$ the Weyl curvature vanishes in the conformally flat Minkowski spacetime. This removes the gravitational degrees of freedom and the entropy S and leads to the second law of thermodynamics as $G(t) = G_N\xi(t)$ grows monotonically from $t = 0$. Fluctuations $\delta\xi$ in the field ξ are defined by

$$\xi(\mathbf{x}, t) = \xi_0(t) + \delta\xi(\mathbf{x}, t), \tag{57}$$

where $\xi_0(t)$ denotes the background FLRW value of ξ. The instability of the Minkowski spacetime in $\mathcal{M}_M$ can lead to $G = G_N\xi$ becoming non-zero and the universe will begin to expand as t increases.

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